Long-wavelength excitations of Higgs condensates

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Abstract

Quite independently of the Goldstone phenomenon, recent lattice data suggest the existence of gap-less modes in the spontaneously broken phase of a $\lambda\Phi^4$ theory. This result is a direct consequence of the quantum nature of the 'Higgs condensate' that cannot be treated as a purely classical c-number field.

Spontaneously broken $\lambda\Phi^4$ theories have been used to fix the ground state in many quantum field theoretical models, including the Standard Model of electroweak interactions. Traditionally, with the notable exception of the Coleman-Weinberg [1] paper, the symmetry breaking mechanism has been searched into a double-well classical potential with perturbative loop corrections. At the same time, in the absence of the Goldstone phenomenon, i.e. for a one-component theory, the particle content of the broken phase is represented as a single massive field, the Higgs boson. In this picture, the lowest excitations of the theory should have an energy spectrum of the form $\tilde{E}(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M_h^2}$ so that the 'Higgs mass' M_h coincides with $\tilde{E}(0)$, the energy-gap of the broken phase.

A possible objection to the above simple description can arise after having tried to represent spontaneous symmetry breaking as a real 'condensation' phenomenon [2]. Although the nature of the elementary condensed quanta is not precisely known, it is conceivable that, beyond the simplest approximation where the 'Higgs condensate' is treated as a classical c-number field, there may be collective effects that can change qualitatively the energy spectrum in the long-wavelength region. As an example, let us consider Landau's quasi-particle picture. This has been proved to be very useful to describe the elementary excitations for a large variety of many-particle systems. However, there are cases where the leading term in the energy spectrum is *not* proportional to $\mathbf{p}^2$ when $\mathbf{p} \to 0$. In these situations, defining an effective quasiparticle mass as for instance ('NR'= Non-Relativistic)

$$\frac{1}{2m_{\text{eff}}} = \lim_{\mathbf{p}\to 0} \frac{\partial E_{\text{NR}}(\mathbf{p})}{\partial \mathbf{p}^2} \tag{1}$$

is misleading. For instance, by using Eq.(1) in a system that has a phonon excitation branch $\sim |\mathbf{p}|$ for $\mathbf{p} \to 0$ we would find an 'effective mass' that becomes smaller and smaller in the limit of a vanishing 3-momentum. The same may occur in the broken phase of $\lambda\Phi^4$ theories where the validity of the identification $M_h = \tilde{E}(0)$ depends on the form of the energy spectrum for $\mathbf{p} \to 0$.

The possibility that the Higgs mass M_h differs non-trivially from the energy-gap of the broken phase has been objectively addressed [3] with lattice simulations. In this case one can precisely measure the exponential decay of the connected correlator at various values of $|\mathbf{p}|$ and determine the energy spectrum of the cutoff theory $\tilde{E}(\mathbf{p})$ by comparing with (the lattice version of) $\sqrt{\mathbf{p}^2 + M_h^2}$.

To exclude possible uninteresting effects, one should preliminarly perform the same analysis in the symmetric phase and compare the corresponding measured energy spectrum $E(\mathbf{p})$ with the form $\sqrt{\mathbf{p}^2 + m^2}$. In the symmetric phase the lattice data [3] give $E(0) = m$ to very high accuracy as expected. However, in the broken phase, $\tilde{E}^2(\mathbf{p}) - \mathbf{p}^2$ turns out to depend

on $|\mathbf{p}|$ when $\mathbf{p} \to 0$ and therefore the attempt to extract M_h from the low-momentum data becomes problematic. If, on the other hand, M_h is consistently extracted from those higher-momentum data where $\tilde{E}^2(\mathbf{p}) - \mathbf{p}^2$ does *not* depend on $|\mathbf{p}|$, then one finds $\tilde{E}(0) < M_h$ with a discrepancy between the two values that seems to increase when taking the continuum limit.

Moreover, new data [4] show that, for the same lattice action, by increasing the lattice size one finds smaller and smaller values of the energy-gap $\tilde{E}(0)$. Namely, by using the same lattice parameters that on a 20^4 lattice give [5] $\tilde{E}(0) = 0.3912 \pm 0.0012$, one finds [4] $\tilde{E}(0) = 0.3791 \pm 0.0035$ on a 24^4 lattice, $\tilde{E}(0) = 0.344 \pm 0.008$ on a 32^4 lattice and $\tilde{E}(0) = 0.298 \pm 0.015$ on a 40^4 lattice. Therefore, differently from M_h which, being extracted from the higher-momentum part of the energy spectrum, shows no dependence on the lattice volume (see Table 2 and Fig.8 of [3]), the energy-gap in the broken phase is an infrared-sensitive quantity that become smaller and smaller by increasing the lattice size and may even vanish in the infinite-volume limit.

An indication in this sense has been obtained in ref.[6] by studying the $p \to 0$ limit of the propagator $G(p)$ after re-summing the infinite series of the zero-momentum tadpole graphs in a given background field. Indeed, the tadpole subgraphs are attached to the other parts of the diagrams through zero-momentum propagators and can be considered a manifestation of the quantum nature of the scalar condensate. In this case, besides the usual massive solution $G^{-1}(0) = M_h^2$, one finds gap-less solutions $G^{-1}(0) = 0$ that would not exist otherwise.

In this Letter, we shall provide another argument in favour of the existence of such gap-less modes. Our discussion is based on the results obtained by Ritschel [7] for the effective potential that we shall first briefly review.

There are two basically different definitions of the effective potential. A first definition is $V_{\rm eff}(\phi) \equiv V_{\rm LT}(\phi)$, i.e. the Legendre transform ('LT') of the generating functional for connected Green's function. In this way $V_{\rm LT}(\phi)$ is rigorously convex downward. For this reason, it is not the same thing as the usual non-convex ('NC') effective potential $V_{\rm eff}(\phi) \equiv V_{\rm NC}(\phi)$ that is found in the ordinary loop expansion. Moreover, in the presence of spontaneous symmetry breaking, $V_{\rm LT}(\phi)$ is *not* an infinitely differentiable function of ϕ [8], differently from $V_{\rm NC}(\phi)$. As shown in ref.[7], the difference between $V_{\rm LT}(\phi)$ and $V_{\rm NC}(\phi)$ amounts to include the quantum effects of the zero-momentum mode that cannot be treated as a purely classical background but requires one more functional integration in field space.

In the following, we shall assume that $V_{\rm NC}(\phi)$ has just a pair $\pm v$ of degenerate absolute minima (the generalization to the more complex situation of several competing minima can be done by using Ritschel's formulas [7]). In addition, we shall assume that $\phi = \pm v$ define

the correct normalization of the background field such that the Higgs mass is defined by the quadratic shape of $V_{\rm NC}(\phi)$ at the minima, namely

$$\left.\frac{d^2 V_{\rm NC}}{d\phi^2}\right|_{\phi=\pm v} \equiv M_h^2 \tag{2}$$

We shall now use Ritschel's results [7] for a space-time constant source J where, after integrating out all non-zero quantum modes, the generating functional is given by

$$Z(J) = \int_{-\infty}^{+\infty} d\phi \;\; \exp[-\Omega(V_{\rm NC}(\phi) - J\phi)] \tag{3}$$

In Eq.(3) Ω denotes the four-dimensional space-time volume and we can define an associated density $w(J)$ as

$$\ln\frac{Z(J)}{Z(0)} \equiv \Omega w(J) \tag{4}$$

In the saddle-point approximation, valid for $\Omega \to \infty$, we get

$$w(J) = \frac{J^2}{2M_h^2} + \frac{\ln\cosh(\Omega J v)}{\Omega} \tag{5}$$

and

$$\varphi = \frac{dw}{dJ} = \frac{J}{M_h^2} + v\tanh(\Omega J v) \tag{6}$$

$$G_J(0) = \frac{d\varphi}{dJ} = \frac{1}{M_h^2} + \frac{\Omega v^2}{\cosh^2(\Omega J v)} \tag{7}$$

Notice that only retaining the full $J-$dependence in Eq.(5) one can fulfill the basic symmetry property

$$\varphi(-J) = -\varphi(J) \tag{8}$$

To determine the zero-momentum propagator in a given background φ, we should now invert J as a function of φ from Eq.(6) and replace it in (7). However, being interested in the limit $J \to 0$ it is easier to look for the possible limiting behaviours of (7).

Since both J and Ω are dimensionful quantities, it is convenient to introduce dimensionless variables

$$x \equiv \Omega J v \tag{9}$$

and

$$y \equiv \Omega v^2 M_h^2 \tag{10}$$

so that Eqs.(6) and (7) become

$$\varphi = v\;[\frac{x}{y} + \tanh(x)] \tag{11}$$

and

$$G_J(0) = \frac{1}{M_h^2}\ [1 + \frac{y}{\cosh^2(x)}] \tag{12}$$

Therefore, the two limits $\Omega \to \infty$ and $J \to \pm 0$ correspond to various paths in the two-dimensional space (x, y). The former gives simply $y \to \infty$. The latter, on the other hand, is equivalent to $\frac{x}{y} \to \pm 0$ since

$$\frac{J}{M_h^2 v} = \frac{x}{y} \tag{13}$$

with many alternative possibilities. If we require a non-zero limit of φ this amounts to an asymptotic non-zero value of x. If this value is finite, say $x = x_o$ we get asymptotically

$$\varphi \to v \tanh(x_o) \tag{14}$$

and

$$G_J(0) \to \frac{y}{M_h^2 \cosh^2(x_o)} \to \infty \tag{15}$$

implying the existence of massless modes for every value of φ. On the other hand, for $x \to \pm\infty$ we obtain

$$\varphi \to \pm v \tag{16}$$

and in this case $G_J(0)$ tend to $\frac{1}{M_h^2}$ or to $+\infty$ depending on whether y diverges slower or faster than $\cosh^2(x)$.

The above results admit a simple geometrical interpretation in terms of the shape of the effective potential $V_{\rm LT}(\varphi)$. After obtaining J as a function of φ from Eq.(6), the inverse zero-momentum propagator in a given background φ is related to the second-derivative of the Legendre-transformed effective potential, namely

$$G_\varphi^{-1}(0) = \frac{dJ}{d\varphi} = \frac{d^2 V_{\rm LT}}{d\varphi^2} \tag{17}$$

In this case, Eqs.(14) and (15) require a vanishing result from Eq.(17) when $-v < \varphi < v$. This is precisely what happens since the Legendre-transformed effective potential becomes flat in the region enclosed by the absolute minima of the non-convex effective potential when $\Omega \to \infty$. This is the usual 'Maxwell construction' where $V_{\rm LT}(\varphi) = V_{\rm NC}(\pm v)$, for $-v \leq \varphi \leq v$, and $V_{\rm LT}(\varphi) = V_{\rm NC}(\varphi)$ for $\varphi^2 > v^2$.

Notice, however, that the limit of (17) for $\varphi \to \pm v$ is different from (2). In fact, identifying the inverse propagators in Eqs. (2) and (17) amounts to a much stronger assumption: the derivative in Eq.(17) has to be a left- (or right-) derivative depending on whether we consider the point $\varphi = -v$ (or $\varphi = +v$). However, this is just a prescription since derivatives depend on the chosen path unless one deals with infinitely differentiable functions.

Therefore, in general, Eq.(17) leads to multiple solutions for the inverse propagator at the absolute minima $\pm v$. Indeed, one can define an exterior derivative for which $G^{-1}_{\rm ext}(0) = M^2_h$ but one also finds $G^{-1}_{\rm int}(0) = 0$, as when approaching the points $\pm v$ from the internal region where the Legendre-transformed potential becomes flat for $\Omega \to \infty$. These two different alternatives correspond to the various limits $y \to \infty$ and $x \to \pm\infty$ in Eq.(12) such that $\frac{y}{\cosh^2(x)}$ tends to zero or infinity.

Now, although determining the finite-momentum propagator requires to study the response to a space-time dependent source, our results allow to draw some definite conclusions. In the broken phase, i.e. for all values of φ that remain non-vanishing when $J \to \pm 0$, there are solutions with $G^{-1}_\varphi(0) = 0$. Therefore, there will always be solutions $G^{-1}_\varphi(p)$ that vanish when $p_\mu \to 0$ implying the existence of gap-less modes whose energy also vanishes when $\mathbf{p} \to 0$. We can express the required relations for such modes in terms of unknown slope parameters η

$$\lim_{\mathbf{p}\to 0} \tilde{E}^2(\mathbf{p}) = \eta \mathbf{p}^2 \tag{18}$$

These describe long-wavelength collective excitations and are a direct consequence of the quantum nature of the 'Higgs condensate'.

Quite independently of the presence of massive modes, the massless modes dominate the propagation over large distances. In the spontaneously broken phase they are clearly visible in the lattice data of ref.[4] where the energy-gap $\tilde{E}(0)$ is found to become smaller and smaller by increasing the lattice size. As such, $\tilde{E}(0)$ cannot represent the operative definition of the Higgs mass M_h. This has to be extracted from the data at larger $|\mathbf{p}|$ that are well reproduced by the single-particle form $\sqrt{\mathbf{p}^2 + \text{const}}$ after checking that the numerical value of the constant under square root does not depend on the lattice size [3, 4]. As mentioned at the beginning, insisting in the identification $\tilde{E}(0) = M_h$ would represent the relativistic analogue of applying Landau's definition of a quasi-particle mass in a system that has a phonon excitation branch for $\mathbf{p} \to 0$.

We conclude remarking that the peculiar infrared behaviour we have pointed out does not depend at any stage on the existence of a continuous symmetry of the classical potential. As such, there should be no differences in a spontaneously broken O(N) theory. Beyond the approximation where the 'Higgs condensate' is treated as a classical c-number field, one has to perform one more integration over the zero-momentum mode of the condensed $\sigma-$field. Therefore, one finds massless solutions by computing the inverse propagator of the $\sigma-$field through Eq.(17) . In this sense, the existence of gap-less modes of the singlet Higgs field does not depend on the number of field components.

References

[1] S. Coleman and E. Weinberg, *Phys. Rev.* **D7**, 1888 (1973).

[2] An intuitive description of the possible physical mechanisms underlying the formation of the broken phase has been given by M. Consoli and P. M. Stevenson, *Int. Journ. Mod. Phys.*, **A15**, 133 (2000).

[3] P. Cea, M. Consoli, L. Cosmai, and P. M. Stevenson, *Mod. Phys. Lett.* **A14**, 1673 (1999); P. Cea, M. Consoli and L. Cosmai, Nucl. Phys. **B** (Proc. Suppl.) 83-84 (2000) 658.

[4] P. Cea, M. Consoli and L. Cosmai, *Lattice measurement of the energy-gap in a spontaneously broken phase*, hep-ph/0101219.

[5] K. Jansen, *et al.*, *Nucl. Phys.* **B322**, 698 (1989).

[6] M. Consoli, *A gap-less mode of the singlet Higgs field*, hep-ph/0101241.

[7] U. Ritschel, *Phys. Lett.* **B318**, 617 (1993).

[8] K. Symanzik, *Comm. Math. Phys.* **16**, 48 (1970).